

## Some consequences of a Higgs triplet

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### Abstract

We consider an extension of the scalar sector of the Standard Model with a single complex Higgs triplet  $X$ . Such extensions are the most economic, model-independent way of generating neutrino masses through triplet interactions. We show that a term like  $a_0 \Phi \Phi X^\dagger$  must be included in the most general potential of such a scenario, in order to avoid a massless neutral physical scalar. We also demonstrate that  $a_0$  must be real, thus ruling out any additional source of CP-violation. We then examine the implications of this term in the mass matrices of the singly- and doubly-charged scalar, neutral scalar and pseudoscalar fields. We find that, for small values of  $a_0/v_2$ , where  $v_2$  is the triplet vev, the spectrum allows the decay of heavier scalars into lighter ones via gauge interactions. For large  $a_0/v_2$ , the doubly-charged, singly-charged and neutral pseudoscalar bosons become practically degenerate, while the even-parity neutral scalars remain considerably lighter, thus emphasizing the possibility of decay of the singly-charged or neutral pseudoscalar states into the neutral scalars. Constraints from the  $\rho$ -parameter are used to find non-trivial limits on the charged Higgs mass depending on  $a_0$ . We also study the couplings of the various physical states in this scenario. For small values of  $|a_0|/v_2$ , we find the lightest neutral scalar field to be triplet-dominated, and thus having extremely suppressed interactions with fermion as well as gauge boson pairs.

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# 1 Introduction

Even though the Standard Model (SM) of electroweak interactions has proven to be enormously successful, it is not obvious that a single Higgs doublet field is responsible for giving masses to the weakly interacting vector bosons and fermions [1]. While fermion masses can arise only through Yukawa couplings with Higgs doublets, gauge bosons can acquire masses from higher representations of  $SU(2)$  as well. Although phenomenological constraints such as that from the  $\rho$ -parameter restrict the vacuum expectation values (vev) of scalar multiplets higher than dimension 2 [2–5], such multiplets are not necessarily without phenomenological significance [6–9]. For example, Higgs triplets can generate Majorana masses for neutrinos once  $\Delta L = 2$  interactions are allowed, thereby avoiding the necessity of right-handed neutrinos [10]. Higgs triplets are also a part of the particle spectrum of some theories attempting stabilization of the electroweak symmetry scale, such as Little Higgs models, even in their relatively economical forms. Higher representations of scalars have some additional phenomenological implications such as  $WZ$  interactions of a singly charged scalar [11].

Extensions of the Higgs sector of the SM employing additional singlet [13], doublet [12, 14–18] as well as triplet fields [2–9] have frequently been considered in the literature. Among these the extensions involving Higgs triplets are particularly interesting, primarily because of their capacity to generate neutrino masses [10], as mentioned above. There are studies in this spirit on left-right symmetric models with or without supersymmetry [19, 20], the Little Higgs models [7], as also on situations with both complex and real scalar triplets whose vevs are related through a custodial symmetry [2, 3, 21]. Some of these scenarios imply rather interesting collider signals that has been at least partially explored in various studies [11, 22–25]. Such studies, as also information extracted on the mass spectrum, help not only in understanding the overall physics of electroweak symmetry breaking (EWSB), but also in probing the scalar potential as a specific component of the theory.

Since it is always helpful to get a model-independent perspective, we consider here a scenario with just the added component for neutrino mass generation, assuming *one doublet* ( $\Phi$ ) and *one complex triplet* ( $X$ ) scalar. In such a case (as opposed to one with complex as well as real triplets, with their vev's related), the vev of the triplet must be relatively small ( $\lesssim 12$  GeV [26]) to satisfy the constraint on the  $\rho$ -parameter (which translates into a constraint arising from tree-level contributions to the electroweak precision variable  $T$ ) [2–5]. The other important oblique parameter, namely,  $S$ , does not provide any serious constraint on this scenario (including the special inputs of this study outlined below), since the mixing of the triplet scalar with the doublet, being proportional to the triplet vev, is small [27].

The salient features of this study, and the new observations arising therefrom, are as follows:

- A term proportional to  $\Phi\Phi X^\dagger$  is retained in the scalar potential and not left out by invoking a discrete symmetry, as has often been done in recent studies [2, 3].
- It is seen that, when there is no real scalar triplet, leaving out the above trilinear term (which adds a dimension-full parameter to the Lagrangian [28]) implies a global  $O(2)$

symmetry in the neutral scalar sector. Giving the neutral fields vev thus results in an additional Goldstone boson, which remains as a physical field, and is inconsistent with experimental observations. *With just one complex triplet added, such an unacceptable situation is avoided only if the trilinear term is retained.*

- It can be seen from very general considerations that the coefficient of the trilinear term, must be real. Thus its introduction does not entail any additional CP-violating phase(s), implying that the scalar sector cannot contain any seed of CP-violation with one doublet and one complex triplet only. As a corollary, we show that more than one multiplet of any given kind is necessary to have CP-violation in the scalar sector.
- The value as well as the sign of the coefficient of the trilinear term is subject to rather non-trivial constraints from the electroweak symmetry breaking conditions, the requirement of the potential bounded from below, and the absence of tachyonic modes.
- The ordering of the scalar spectrum and the composition of the lightest neutral scalar depend on the coefficient of the trilinear term, as demonstrated using several benchmark values of different parameters occurring in the model. This in turn affects the fermionic and gauge couplings of the low-lying physical states, and restricts the viability of different decay chains of the relatively heavier states.

The field content and the structure of the potential have been outlined in Section 2, where we have also discussed why such a potential cannot lead to CP-violating effects. In Section 3, we compute the masses of the scalar fields, and in Section 4, various constraints on the scalar potential are discussed. Taking all these constraints into account, we show, in Section 5, the mass spectrum as well as the couplings of the scalar fields to fermions and gauge bosons, where we also discuss their overall implications. We summarize and conclude in Section 6.

## 2 The triplet Higgs model

In the simple model that we consider here, the Higgs sector consists of a complex scalar  $Y = 2$  triplet  $X$ , along with the usual complex  $Y = 1$  doublet  $\Phi$  of the SM:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad X = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^{0*} \end{pmatrix}. \quad (1)$$

We choose phase conventions for the fields such that  $(\chi^{++}, \chi^+, \phi^+)^* = (\chi^{--}, \chi^-, \phi^-)$ , and assign vevs to the neutral components as follows:

$$\langle \phi^0 \rangle = v_1/\sqrt{2}, \quad \langle \chi^0 \rangle = v_2. \quad (2)$$

Moreover, we can use the freedom of choosing the relative phase of  $X$  and  $\Phi$  and align  $v_1/\sqrt{2}$  and  $v_2$  simultaneously along the real axis without any loss of generality. The most general

potential for the scalar sector can be written as  $V_{\text{total}} = V_2 + V_3 + V_4$ , where the subscript attached to each term stands for the number of fields occurring in it. Individually,

$$V_2 = -\mu_1^2 (\Phi^\dagger \Phi) + \mu_2^2 (X^\dagger X) \quad (3)$$

$$V_3 = \sqrt{3}a_0 (\Phi \Phi X^\dagger) + \text{h.c.} \quad (4)$$

$$V_4 = \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (X^\dagger X)^2 + \lambda_3 (\Phi^\dagger \Phi) (X^\dagger X) + \lambda_4 (\Phi^\dagger \tau_i \Phi) (X^\dagger t_i X) + \lambda_5 |X^T C X|^2, \quad (5)$$

where

$$C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (6)$$

and  $\tau_i$ s and  $t_i$ s ( $i = 1-3$ ) are the Pauli matrices in 2 and 3 dimensions respectively. The factor of  $\sqrt{3}$  in  $V_3$  is taken for later convenience. Note that in the bilinear term  $V_2$ , we put a ‘wrong sign’ for the mass term for the doublet fields for spontaneous symmetry breaking to take place, while we put a ‘correct sign’ for the mass term for the triplet fields. This is required for keeping the triplet vev naturally small [7], which in turn is needed for avoiding large corrections to the  $\rho$ -parameter. This choice ensures that the triplet vev arises only through the trilinear and quartic terms, and can remain small without requiring the triplet mass(es) to be below acceptable limits.

The trilinear term  $V_3$  has often been neglected in the literature [2, 3]. One way to do this is to demand the potential to be invariant under the discrete transformations  $\Phi \rightarrow -\Phi$  and  $X \rightarrow -X$ . However, as we mentioned earlier, imposing such a discrete symmetry has no definite theoretical motivation. In particular, such a discrete symmetry in any case needs to be abandoned in the fermion interaction terms, if one thinks in terms of neutrino Majorana masses generated with a Higgs triplet [10]. Furthermore, a term of the form  $\Phi \Phi X^\dagger$  bears a close analogy to one like  $\ell \ell X$ , as far as gauge structure is concerned. Since the latter is an indispensable part of the Type II seesaw mechanism for the generation of neutrino masses, retaining the trilinear scalar term seems to be quite natural. In addition, it also helps one in understanding the smallness of the triplet vev, by postulating a positive mass-squared term for the triplet, and letting it develop a vev through doublet-triplet mixing only (second reference of [7]).

Keeping this term also enables us to ward off an additional undesirable Goldstone boson, as will be shown in the next section. Though this term could in principle have a complex coefficient, which might indicate an extra source of CP-violation, we shall show below that actually the parameter  $a_0$  has to be real.

The third and fourth terms of  $V_4$  represent two singlet states that can be constructed out of two  $\Phi$  fields and two  $X$  fields. The product  $\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{3} \otimes \mathbf{3}$  of SU(2) contains two mutually orthogonal singlet combinations. Any other combination, like the singlet coming out of  $(\Phi^\dagger X)(X^\dagger \Phi)$ , can be expressed in terms of these two singlets.

We expand the neutral components of both  $X$  and  $\Phi$  about their vevs, and write

$$\phi^0 = \frac{1}{\sqrt{2}} (\phi^{0R} + v_1 + i\phi^{0I}), \quad \chi^0 = \frac{1}{\sqrt{2}} (\chi^{0R} + \sqrt{2}v_2 + i\chi^{0I}). \quad (7)$$

Next, using  $\Phi^\dagger\Phi = (\phi^+\phi^- + \phi^0\phi^{0*})$ , and  $X^\dagger X = (\chi^{++}\chi^{--} + \chi^+\chi^- + \chi^0\chi^{0*})$ , we express all the terms in  $V$  in terms of the component fields:

$$\begin{aligned}
V_2 &= -\mu_1^2 [\phi^+\phi^- + \phi^0\phi^{0*}] + \mu_2^2 [\chi^{++}\chi^{--} + \chi^+\chi^- + \chi^0\chi^{0*}], \\
V_3 &= a_0 \left[ \phi^0\phi^0\chi^0 - \sqrt{2}\phi^+\phi^0\chi^- + \phi^+\phi^+\chi^{--} \right] + \text{h.c.}, \\
V_4 &= \lambda_1 [\phi^+\phi^-\phi^+\phi^- + \phi^0\phi^{0*}\phi^0\phi^{0*} + 2\phi^+\phi^-\phi^0\phi^{0*}] \\
&\quad + \lambda_2 [\chi^{++}\chi^{--}\chi^{++}\chi^{--} + \chi^+\chi^-\chi^+\chi^- + \chi^0\chi^{0*}\chi^0\chi^{0*} + 2\chi^{++}\chi^{--}\chi^0\chi^{0*} \\
&\quad + 2\chi^+\chi^-\chi^0\chi^{0*} + 2\chi^{++}\chi^{--}\chi^+\chi^-] \\
&\quad + \lambda_3 [-\chi^{++}\chi^{--}\phi^+\phi^- + \chi^{++}\chi^{--}\phi^0\phi^{0*} + \phi^+\phi^-\chi^+\chi^- + \phi^+\phi^-\chi^0\chi^{0*} \\
&\quad + \chi^+\chi^-\phi^0\phi^{0*} + \phi^0\phi^{0*}\chi^0\chi^{0*}] \\
&\quad + \lambda_4 [\chi^{++}\chi^{--}\phi^+\phi^- + \chi^{++}\chi^{--}\phi^0\phi^{0*} - \phi^+\phi^-\chi^0\chi^{0*} + \phi^0\phi^{0*}\chi^0\chi^{0*} \\
&\quad + \{\sqrt{2}\phi^+\chi^-\phi^{0*}\chi^{0*} + \sqrt{2}\phi^+\chi^+\chi^{--}\phi^{0*} + \text{h.c.}\}] \\
&\quad + \lambda_5 [4\chi^{++}\chi^{--}\chi^0\chi^{0*} + \chi^+\chi^-\chi^+\chi^- - 2\{\chi^+\chi^+\chi^{--}\chi^0 + \text{h.c.}\}]. \tag{8}
\end{aligned}$$

In the limit  $a_0 = 0$ , the neutral sector of the above potential has an additional  $O(2)$  symmetry, which keeps it invariant under a rotation in the  $(\phi^{0R}, \phi^{0I})$  and  $(\chi^{0R}, \chi^{0I})$  planes. Thus an extra neutral massless Goldstone boson, over and above the one arising from the breaking of  $SU(2) \times U(1)$ , arises when  $\Phi$  and  $X$  acquire vevs, making the scenario phenomenologically unacceptable. This problem can be avoided if one also introduces a real scalar triplet [2–4]. One therefore concludes that the trilinear term  $V_3$ , written in terms of the dimension-full parameter  $a_0$ , must be there in the potential if one has a complex triplet and the usual doublet. The role of  $a_0$  in determining the spectrum of physical states and their coupling to fermion or gauge boson pairs is thus of considerable importance, if one has to understand the phenomenology of this ‘most economical’ scenario involving a scalar triplet.

Is there a possibility of CP violation with a possibly complex  $a_0$ ? The answer is in the negative, since one can always end up with the vevs  $v_1$  and  $v_2$  aligned without any loss of generality. All the  $\lambda$ s must be real, since the quartic field combinations are self-hermitian. Even if we start with a complex  $a_0$ ,  $V_3$  as in equation (8) can be made real by absorbing the phase in the field combination  $\Phi\Phi X^\dagger$ , thus removing any chance of CP violation. This is in contrast to, say, the most general scenario with two Higgs doublets  $\Phi_1$  and  $\Phi_2$ , where the freedom of adjusting the relative phase between the two doublets does not rotate away the CP-violating phases of terms of the form  $\Phi_1^\dagger\Phi_2$  and  $(\Phi_1^\dagger\Phi_2)^2$  at the same time. In our case, the relative phase of  $\Phi$  and  $X$  shows up only in the term proportional to  $a_0$ , and can therefore be adjusted to render  $a_0$  real.

The above argument also applies to a model containing one doublet  $\Phi$ , one complex triplet  $X$  and a real triplet  $\Psi$  [2, 3]. In this case, there can be two relative phases among the three multiplets. At the same time, there are only two terms in the most general potential where the relative phases may occur explicitly, namely, those proportional to  $\Phi\Phi X^\dagger$  and  $\Phi\Phi X^\dagger\Psi$  [12]. Obviously, the freedom of the two relative phases can be used to rotate away any phases in the coefficients of both the above terms. Thus the tree-level potential does not allow CP-violation with Higgs doublets, complex triplets and real triplets so long one has *not more than one of a particular kind* of multiplets.

### 3 Masses and couplings of the physical scalars

We have mentioned that the trilinear term affects the mass spectrum. This also alters the composition of the physical states, and changes the various constraints on the potential.

Minimization of the most general potential in our scenario leads to the following conditions:

$$\begin{aligned} -\mu_1^2 + v_1^2 \lambda_1 + v_2^2 [\lambda_3 + \lambda_4] + 2v_2^2 \delta &= 0, \\ \mu_2^2 + 2v_2^2 \lambda_2 + \frac{1}{2}v_1^2 [\lambda_3 + \lambda_4] + \frac{1}{2}v_1^2 \delta &= 0, \end{aligned} \quad (9)$$

where we have introduced the dimensionless parameter  $\delta$ , defined as

$$\delta = \frac{a_0}{v_2}. \quad (10)$$

To start with, the general potential considered by us has 10 parameters: two vevs, two  $\mu$ 's, five  $\lambda$ s and  $a_0$ . Making use of the two potential minimization conditions the number of independent parameters have been reduced to 8, since the two  $\mu$ 's can be expressed in terms of the other parameters.

#### 3.1 Mass of the doubly-charged field

After collecting the coefficients of  $\chi^{++}\chi^{--}$  terms from the total potential, replacing the neutral fields with the respective vevs, and applying the minimization conditions on the potential, we obtain

$$M_{H^{\pm\pm}}^2 = 4v_2^2 \lambda_5 - v_1^2 \left( \lambda_4 + \frac{1}{2}\delta \right), \quad (11)$$

To avoid tachyonic scalars, we should take either  $\lambda_4$  or  $\delta$  to be negative. We will see later that the negativity of  $\delta$  is forced by the neutral pseudoscalar mass matrix. From now, we will denote the doubly charged mass eigenstate by  $H^{\pm\pm}$ , which, in this model, is identical with  $\chi^{\pm\pm}$ .

#### 3.2 Masses of the singly-charged fields

The mass-squared matrix for the singly charged fields is

$$\mathcal{M}_{\pm}^2 = - \begin{pmatrix} 2(\lambda_4 + \delta)v_2^2 & -(\lambda_4 + \delta)v_1v_2 \\ -(\lambda_4 + \delta)v_1v_2 & \frac{1}{2}(\lambda_4 + \delta)v_1^2 \end{pmatrix}, \quad (12)$$

whose two eigenvalues are

$$0, \quad -\frac{1}{2}v^2(\lambda_4 + \delta), \quad (13)$$

where  $v = \sqrt{v_1^2 + 4v_2^2}$ . The respective charged scalar mass eigenstates turn out to be

$$G^\pm = \frac{v_1}{v}\phi^\pm + \frac{2v_2}{v}\chi^\pm \quad (14)$$

$$H^\pm = \frac{-2v_2}{v}\phi^\pm + \frac{v_1}{v}\chi^\pm \quad (15)$$

It is important to note the following points:

- Doublet-triplet mixing does not depend on the parameter  $a_0$ , and is also small (since  $v_2 \ll v_1$ ).
- The mass of  $H^\pm$  can be significantly large as it depends on the ratio  $a_0/v_2$ , where typically  $a_0$  can be  $\sim 1$  TeV and  $v_2$  is small. Thus the effect of the trilinear term is mainly to push up the mass of the dominantly triplet state, without changing its constitution, so that this state almost decouples from low-energy theory for a very high value of  $a_0/v_2$ .
- In general, the doubly charged and singly charged mass eigenstates are non-degenerate (even when both  $a_0$  and  $\lambda_5$  vanish). This is because the lifting of degeneracy through SU(2) breaking can be driven by  $v_1$ , the electroweak scale vev.

### 3.3 Masses of the neutral fields

The neutral scalar and pseudoscalar mass matrices in this scenario are

$$\mathcal{M}^{0R} = \begin{pmatrix} v_1^2\lambda_1 & B \\ B & 2v_2^2\lambda_2 - v_1^2\delta/4 \end{pmatrix}, \quad \mathcal{M}^{0I} = -\delta \begin{pmatrix} 2v_2^2 & \frac{1}{\sqrt{2}}v_1v_2 \\ \frac{1}{\sqrt{2}}v_1v_2 & \frac{1}{4}v_1^2 \end{pmatrix}. \quad (16)$$

where  $B = v_1v_2[\lambda_3 + \lambda_4 + \delta]/\sqrt{2}$ . Two massive even-parity physical states, denoted as  $h^0$  and  $H^0$ , are obtained upon diagonalizing  $\mathcal{M}^{0R}$ . We shall comment on the masses and compositions of these states later in this section.

$\mathcal{M}^{0I}$ , on the other hand, has one massive physical state  $A^0$ , the other state being the neutral Goldstone boson  $G^0$ . The eigenvalues of  $\mathcal{M}^{0I}$  are

$$0, \quad -\frac{a_0}{4v_2}(v_1^2 + 8v_2^2), \quad (17)$$

whence the pseudoscalar physical state emerges as

$$A^0 = \frac{2\sqrt{2}v_2}{\sqrt{v_1^2 + 8v_2^2}}\phi^{0I} + \frac{v_1}{\sqrt{v_1^2 + 8v_2^2}}\chi^{0I} \quad (18)$$

with a mass-squared value given by  $-\frac{1}{2}\delta(v_1^2 + 8v_2^2)$ .

The following observations can be made on the mass eigenstates in the neutral sector:

- $\delta$ , and hence  $a_0$ , must be *negative* to avoid a tachyonic scalar.
- Without the  $a_0$  term, the model is plagued with the additional Goldstone boson  $A^0$ . The root of this lies in an additional  $O(2)$  symmetry in the neutral sector, which is broken explicitly when  $a_0$  is non-zero. Such a symmetry could have been there in the charged sector as well, but for the term  $(\Phi^\dagger \tau_i \Phi)(X^\dagger t_i X)$ . This would have led to a massless scalar having  $SU(2)$  gauge couplings. Since the experimental observations on  $Z$ -decay disallows such a scalar, *the trilinear term driven by  $a_0$ , which breaks the  $O(2)$  explicitly, is therefore a necessary requirement of a model where there is just an  $SU(2)$  doublet and a complex triplet* (the simultaneous presence of a real triplet breaks this  $O(2)$ , and thus a trilinear term is avoidable in the potential of such a model).
- As in the charged scalar sector, doublet-triplet mixing in the pseudoscalar sector does not depend on the parameter  $a_0$ .

## 4 Additional constraints on the potential

There are some additional constraints on the remaining parameters, arising from the demand that (a) all the physical scalars must have non-negative mass-squared values, (b) the potential has to be bounded from below, and (c)  $V_{\min} < 0$  for spontaneous symmetry breaking.

### 4.1 Absence of tachyonic modes

From the requirement that all the eigenvalues of the mass-squared matrices should be positive, one obtains the following conditions:

For the doubly charged field

$$4v_2^2 \lambda_5 - v_1^2 \left( \lambda_4 + \frac{1}{2} \delta \right) > 0, \quad (19)$$

for the singly charged fields

$$-\frac{1}{2} v^2 (\lambda_4 + \delta) > 0, \quad (20)$$

and for the physical neutral pseudoscalar field

$$a_0 < 0. \quad (21)$$

Since  $v_2 \ll v$ , we can drop the first term of equation (19) as a first approximation, which in turn gives the stronger constraint

$$\lambda_4 < -\frac{1}{2} \delta. \quad (22)$$



## 4.2 Boundedness from below

In order to examine the boundedness of the potential from below, we first extract the part of the potential involving neutral fields only:

$$V = -\mu_1^2 (\phi^0 \phi^{0*}) + \mu_2^2 (\chi^0 \chi^{0*}) + \lambda_1 (\phi^0 \phi^{0*})^2 + \lambda_2 (\chi^0 \chi^{0*})^2 + \lambda_3 (\phi^0 \phi^{0*}) (\chi^0 \chi^{0*}) + \lambda_4 (\phi^0 \phi^{0*}) (\chi^0 \chi^{0*}) + a_0 (\phi^0 \phi^0 \chi^0 + \text{h.c.}) . \quad (23)$$

The following conditions follow from the requirement of boundedness from below:

1. Since the terms quartic in fields are dominant ones, positivity of  $\lambda_1$  and  $\lambda_2$  ensures that the potential is bounded from below in absence of any coupling between doublet and triplet fields.
2. The third, fourth, fifth and sixth terms of the potential together can be written as

$$\left[ \sqrt{\lambda_1} (\phi^0 \phi^{0*}) - \sqrt{\lambda_2} (\chi^0 \chi^{0*}) \right]^2 + \left( \lambda_3 + \lambda_4 + 2\sqrt{\lambda_1 \lambda_2} \right) (\phi^0 \phi^{0*}) (\chi^0 \chi^{0*}) , \quad (24)$$

where the first term is non-negative and vanishes in the direction  $|\phi^0|^2/|\chi^0|^2 = \sqrt{\lambda_2/\lambda_1}$ . In that case, in order that the potential is bounded from below, one requires

$$\left( \lambda_3 + \lambda_4 + 2\sqrt{\lambda_1 \lambda_2} \right) > 0 . \quad (25)$$

3. One can still have potentially dangerous directions in which the combined contribution of the quartic terms vanishes. However, a delineation of conditions ensuing from this requires the computation of higher-order corrections to the potential, which can yield conditions on  $a_0$  for either a potential bounded from below, or a false vacuum whose lifetime exceeds the age of the universe [29].

## 4.3 $V_{\min} < 0$

To implement this condition, we start from the total potential keeping neutral fields only and replace them by their vevs, to obtain

$$f(v_1, v_2) = -\frac{1}{2}\mu_1^2 v_1^2 + \mu_2^2 v_2^2 + a_0 v_1^2 v_2 + \frac{1}{4}\lambda_1 v_1^4 + \lambda_2 v_2^4 + \frac{1}{2}[\lambda_3 + \lambda_4] v_1^2 v_2^2 . \quad (26)$$

Substituting  $\mu_1^2$  and  $\mu_2^2$  from equation (9) in  $f(v_1, v_2)$ , we get

$$f_{\min} = -\frac{1}{4}\lambda_1 v_1^4 - \lambda_2 v_2^4 - \frac{1}{2}(\lambda_3 + \lambda_4 + \delta) v_1^2 v_2^2 . \quad (27)$$

The negativity of the potential at the minimum thus requires

$$|\delta| < \left( \frac{1}{2}\lambda_1 \frac{v_1^2}{v_2^2} + 2\lambda_2 \frac{v_2^2}{v_1^2} + \lambda_3 + \lambda_4 \right) , \quad (28)$$

which, to the leading order, can be written as,

$$|\delta| < \frac{1}{2}\lambda_1 \left(\frac{v_1}{v_2}\right)^2. \quad (29)$$

Two more conditions for  $V_{\min}$  come from the requirements  $\partial^2 f / \partial v_1^2 > 0$  and  $\partial^2 f / \partial v_2^2 > 0$ , which can simply be written as  $\mathcal{M}_{11}^{0R}, \mathcal{M}_{22}^{0R} > 0$ .

We observe from equation (29) that not only does  $a_0$  have to be negative, but is also bounded above, given the constraint on the magnitude of  $v_2$ . However, the upper limit on  $a_0$  can be way above the TeV scale if  $v_2$  is very small. We shall see later that the magnitude of  $a_0$  is subject to further constraints, which avoid this possibility.

The allowed region in the parameter space of this model has to satisfy all of the above conditions. They have been taken into account in the numerical studies on the mass spectrum and the strengths of various couplings, reported in the next section.

## 5 Some numerical results

In the following two sections we talk about the numerical values of masses of the physical scalar fields as well as their couplings to the fermions and gauge bosons, and discuss their overall implications.

### 5.1 Mass spectrum: numerical values

Let us first remind ourselves of the roles played by various parameters in the scalar potential in determining the mass spectrum of the model. A clear idea of this is obtained from equations (11), (13), (16), and (17). On eliminating the mass parameters  $\mu_1$  and  $\mu_2$  from the EWSB conditions equation (9), the physical masses are completely determined by the two vevs  $v_1$  and  $v_2$ ,  $\lambda_i$  ( $i = 1-5$ ), as well as by the dimensionless quantity  $\delta$ . The scale of the doublet-dominated neutral scalar is set by  $\lambda_1$ .  $\lambda_2$  affects the masses at the level of  $v_2^2$  only, while  $\lambda_3$  does not appear in the expressions for masses, after using the EWSB conditions.  $\lambda_5$  only affects the doubly charged scalar masses to order  $v_2^2$ .

For small  $|\delta|$ , the masses of the states  $H^+$  and  $H^{++}$  depend on  $\lambda_4$  and  $\delta$ , while the mass of  $A^0$  depends on  $\delta$  alone (apart from the vevs). In addition, the requirement of making the quartic terms gauge invariant makes the different mass matrices dependent on the SU(2) Clebsch-Gordan coefficients. The mass expressions clearly show that for positive  $\lambda_4$ ,  $M_{A^0} > M_{H^+} > M_{H^{++}}$ , while for negative  $\lambda_4$ ,  $M_{A^0} < M_{H^+} < M_{H^{++}}$ . This can also be seen in figure 1(a), where we have varied the ratio  $|\delta|$  between 1 and 100,  $v_2$  has been set at 1 GeV, and all the other  $\lambda$ s have been fixed at 1. For small  $|\delta|$ , there is substantial separation in the masses of  $A^0$ ,  $H^+$  and  $H^{++}$ . This has interesting implications from the viewpoint of accelerator phenomenology,

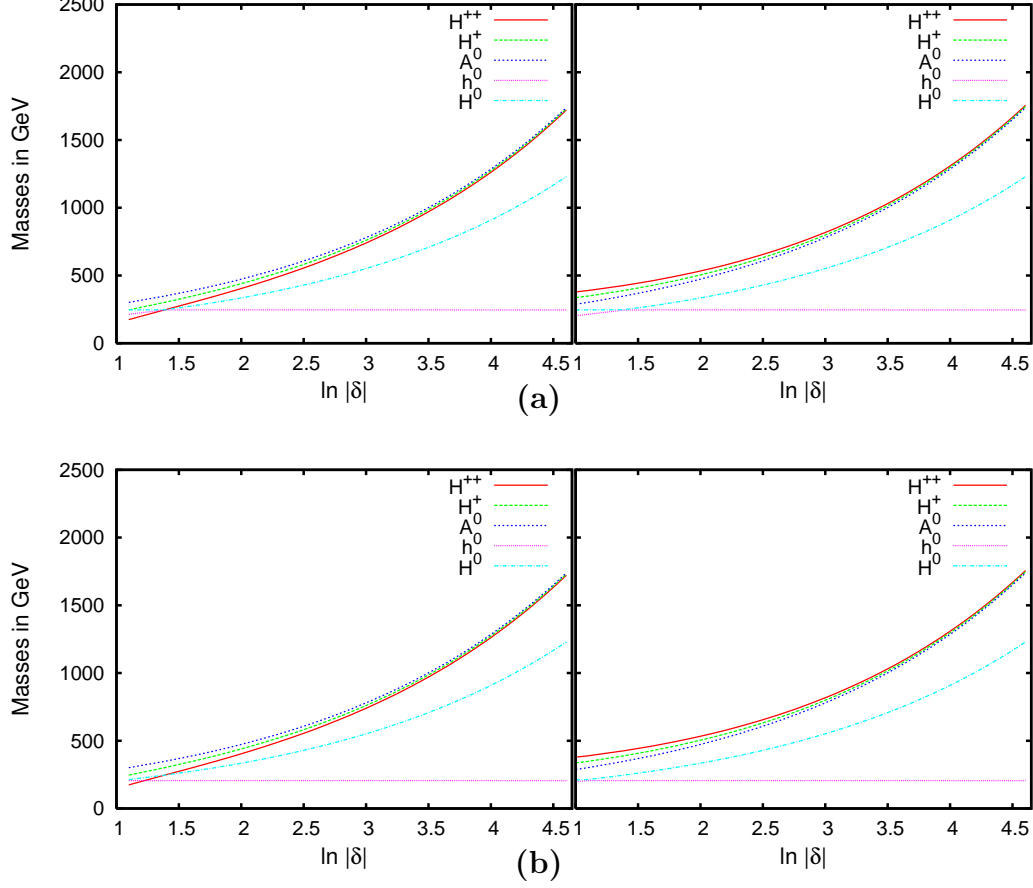


Figure 1: Masses of the scalar physical states as functions of  $\ln|\delta|$  ( $|\delta| = |a_0/v_2|$ ). Top left panel:  $\lambda_i$  ( $i = 1 - 5$ ) = 1,  $v_2 = 1$  GeV, and  $|\delta|$  varies between 1 and 100, resulting in  $M_{A^0} > M_{H^+} > M_{H^{++}}$ . Top right panel: identical with left but only  $\lambda_4 = -1$ , resulting in  $M_{A^0} < M_{H^+} < M_{H^{++}}$ . Bottom left panel:  $\lambda_1 = 0.7$ ,  $\lambda_i$  ( $i = 2 - 5$ ) = 1,  $v_2 = 1$  GeV, and  $|\delta|$  varies between 1 and 100, resulting in  $M_{A^0} > M_{H^+} > M_{H^{++}}$ . Bottom right panel: identical with bottom left but only  $\lambda_4 = -1$ , resulting in  $M_{A^0} < M_{H^+} < M_{H^{++}}$ . In all these figures the lighter neutral scalar  $h^0$  remains triplet-dominated below the cross-over point, around  $\ln|\delta| \simeq 1.39$  for top left and top right panels and around  $\ln|\delta| \simeq 1.03$  for bottom left and bottom right panels, beyond which it is doublet-dominated. It is just the opposite for  $H^0$ .

since the heavier scalars can decay into the lighter ones via gauge couplings in this situation, as for example  $H^{++} \rightarrow H^+W^+$ ,  $H^+ \rightarrow H^0W^+$ ,  $H^+ \rightarrow h^0W^+$  and the like.

Figure 1(a) also shows that  $h^0$  can become very light for small  $\delta$ , for some specific combination of parameters such as  $\lambda_3 = -\lambda_4$ . In such a case, a very light, triplet dominated neutral scalar may exist, evading the limit from  $e^+e^- \rightarrow Zh^0$ . Such a scalar, however, can still be probed, for example, through  $h^0A^0$  production in  $e^+e^-$  collision, or via the  $W^+W^-h^0h^0$  coupling at the LHC (in the gauge boson fusion channel). We do not discuss the phenomenology of such a situation here, especially because the stability of such a small  $h^0$  mass against radiative corrections is yet to be demonstrated. It is also to be noted that in the limit  $a_0 \rightarrow 0$ ,  $M_{A^0}$  approaches zero, because of the emergence of the global  $O(2)$  symmetry in the neutral sector, as discussed in the subsection 3.3.

As  $|\delta|$  grows, the  $A^0$ ,  $H^+$ ,  $H^{++}$  states become degenerate, because the corresponding mass eigenvalues are controlled more and more by the term driven by  $a_0$ . However, the increasingly common value of these three masses stays above those of the heavier CP-even neutral scalars  $H^0$  and  $h^0$ . Again, this creates a lot of opportunity for the heavy scalars being produced from the charged ones through gauge interactions. All the above features remain identical for figure 1(b), for which the parameters are set identical to those in figure 1(a) except for  $\lambda_1 = 0.7$  (the implication of which is explained later in this section).

One should remember that the requirement of neutrino mass generation often leads to the choice of a much smaller  $v_2$  than 1 GeV [10]. While the mass eigenvalues depend only upon  $\delta = |a_0|/v_2$ , it should be noted that a value of  $v_2$  as small as 1 eV will allow very large values of  $\delta$ , if  $a_0$  is around the electroweak scale. An extrapolation of the plots in figure 1 reveals that, in such a case, one has an increasingly wide separation of the neutral scalars with the pseudoscalar as well as the singly- and doubly-charged states. The possibility of the heavier states decaying dominantly into the lighter ones gets further accentuated in such a situation. However, since the physical states are of increasingly ‘pure’ doublets and triplets in this region, such decay is mostly confined to the channels  $H^+ \rightarrow W^+H^0$  and  $A^0 \rightarrow Zh^0$ .

Although the physical charged Higgs mass  $M_{H^+}$  for a given  $M_{h^0}$  depends on  $\lambda_4$  and  $\delta$ , it is still possible to constrain regions in the  $M_{H^+} - a_0$  space from precision data. Such constraint comes essentially from the tree-level upper limit on the  $\rho$ -parameter, which in turn implies an upper limit on  $v_2$  [26]. Using this upper limit (as obtained from the oblique parameter  $T$ ), and at the same time varying  $\lambda_4$  over a range of admissible values (see equation (13)), one thus obtains a minimum value of  $M_{H^+}$  for every  $a_0$ . The parameter space is further restricted by LEP results, and the constraints on the potential discussed in section 4 (see in particular, equation (29)). Note that such constraints may differ for different values of  $M_{h^0}$  (decided by  $\lambda_1$ ), even if one scans over the entire allowed range of other  $\lambda$ ’s. The consequently allowed regions of the  $M_{H^+} - a_0$  space, for  $v_2 < 12$  GeV and two values of  $M_{h^0}$  (120 GeV and 400 GeV) are shown in figure 2 (left and right panels respectively). It is evident from these figures that allowed lower limits on  $M_{H^+}$  can be significantly different from what they would have been for  $a_0 = 0$ <sup>1</sup>. It should be noted that a minimum value of  $a_0$  is obtained; this is because the mass of

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<sup>1</sup>As discussed in section 1, one gets relatively weaker constraints from loop-induced effects. Also, the con-

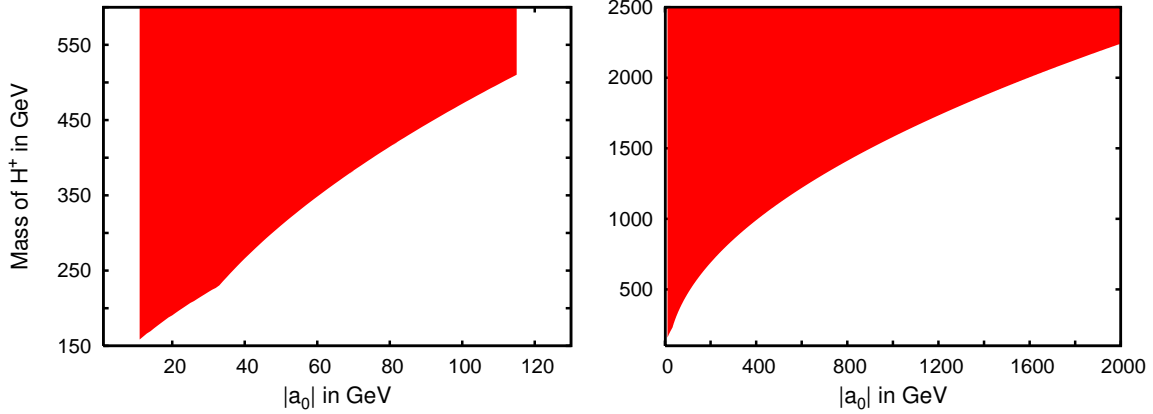


Figure 2: The allowed regions of  $M_{H^+} - a_0$  space, for  $v_2 < 12$  GeV (coming from tree-level upper limit on the  $\rho$ -parameter), and  $M_{h^0} = 120$  GeV (left panel) and  $M_{h^0} = 400$  GeV (right panel), are shown by the shaded region marked by red-color. The constraints restricting this parameter space also include those coming from LEP results as well as from the scalar potential (see section 4)

the ‘physical’ pseudoscalar state is proportional to  $\sqrt{|a_0|}$ , a fact connected with the appearance of a glodstine boson in the limit of vanishing  $a_0$ . Here we have assumed a lower limit of 100 GeV on the pseudoscalar mass, which leads to a minimum allowed value of  $a_0$ . The minimum value pertains to both panels in the figure, though the larger range of  $a_0$  in the right-hand panel makes it nearly invisible.

Also note in both the left and the right panels of figure 1 that the lighter neutral scalar  $h^0$  remains triplet-dominated below a cross-over point decided by  $|\delta|$ , beyond which it is doublet-dominated. For the parameter values we have chosen, this is around  $|\delta| \sim 4$  for figure 1(a), and 2.8 for figure 1(b). These cross-overs are more clearly presented in figure 3, where we have shown the composition of  $h^0$  and  $H^0$  in terms of the probability of it being an SU(2) doublet, as a function of  $|\delta|$  for  $\lambda_4 = 1$ . This can be easily understood from a first-order approximation of  $\mathcal{M}^{0R}$  in equation (16), where we drop the relatively smaller off-diagonal terms and the  $v_2^2$  dependent term of  $\mathcal{M}_{22}^{0R}$ . Since, for figures 3(a) for example, we have taken  $\lambda_1 = 1$ , one has  $\mathcal{M}_{11}^{0R} > \mathcal{M}_{22}^{0R}$  as long as  $|\delta| < 4$ . Thus the lighter neutral scalar will be triplet-dominated, its mass-squared value being approximately given by  $\mathcal{M}_{22}^{0R}$ . For both the plots of figure 3(a), beyond this cross-over,  $\mathcal{M}_{22}^{0R} > \mathcal{M}_{11}^{0R}$ , and the lighter neutral scalar becomes doublet-dominated with  $\mathcal{M}_{11}^{0R}$  as its approximate mass-squared value, and its mass no longer depends on the precise value of  $a_0$ . Following similar reasons, for both the plots of figure 3(b), where we have taken  $\lambda_1 = 0.7$ , the cross-over takes place around  $|\delta| \sim 2.8$ .

Note that for small values of  $v_2$  (see the plots in the right panel of figure 3 where  $v_2 = 1$  eV), this

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straints on the potential may change on considering loop effects, which are tentatively assumed to be small, since the triplet states have no coupling with heavy quarks.

cross-over is less smooth compared to the large  $v_2$  cases (plots in the left panel of figure 3). This is expected since, for small  $v_2$ , the first-hand approximation that we have made above is more exact, making  $\mathcal{M}^{0R}$  almost diagonal. Thus, the transition from  $\mathcal{M}_{11}^{0R} > \mathcal{M}_{22}^{0R}$  to  $\mathcal{M}_{22}^{0R} > \mathcal{M}_{11}^{0R}$  is rather sharp at  $|\delta| = 4\lambda_1$  (see equation (16)). The very small value of  $v_2$  makes the off-diagonal terms of  $\mathcal{M}^{0R}$  inconsequential in rendering the transition somewhat gradual.

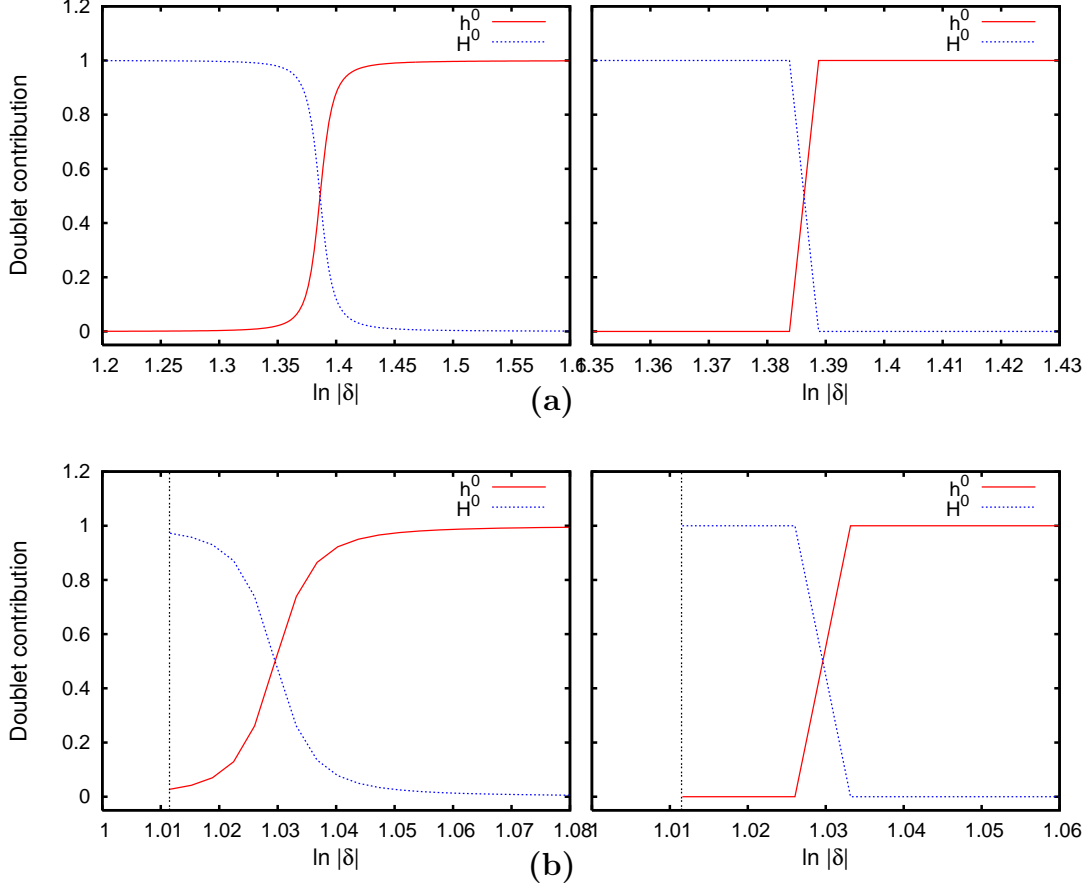


Figure 3: The doublet compositions of  $h^0$  and  $H^0$  as functions of  $\ln|\delta|$ . Top left panel:  $\lambda_i$  ( $i = 1 - 5$ ) = 1 and  $v_2 = 1$  GeV. Top right panel: identical with top left but only  $v_2 = 1$  eV. Bottom left panel: identical with top left but only  $\lambda_1 = 0.7$ . Bottom right panel: identical with top left but  $v_2 = 1$  eV,  $\lambda_1 = 0.7$ . For all cases,  $h^0$  remains triplet-dominated below the cross-over point,  $\ln|\delta| \simeq 1.39$  for top panels and  $\ln|\delta| \simeq 1.03$  for bottom panels, beyond which it is doublet-dominated. Just the opposite happens to  $H^0$ . The regions on the left of the vertical line shown in the figures of 2(b) are disallowed, by the chosen lower limits on the scalar masses, such as,  $M_{H^{++}} > 150$  GeV,  $M_{H^+}$ ,  $M_{A^0}$ ,  $M_{H^0} > 100$  GeV and  $M_{h^0} > 115$  GeV.

## 5.2 Fermion and gauge-boson pair couplings

Among the various possible fermion and gauge-boson pair couplings with the physical scalar states, only the couplings with neutral scalars  $H^0$  and  $h^0$  show dependence on  $a_0$  (see the Appendix for a complete list of all fermion/gauge-scalar couplings of this model). As is obvious from the plots in figure 4, all of  $t$ ,  $b$ ,  $W^\pm$  and  $Z$  pair couplings show the effect of the cross-over in the composition of  $H^0$  and  $h^0$  discussed in the previous section. These plots have been generated for  $|\delta|$  in the range 1-6,  $v_2$  set at 1 GeV, and the  $\lambda_i$  ( $i = 2-5$ ) fixed at 1,  $\lambda_1$  has been set at 1 for the left panel, and at 0.7 for the right panel. We do not show the corresponding plots for other benchmark points here (i. e. small  $v_2$ , negative  $\lambda_4$ ), since they *do not* add additional information to the general observations made below.

For higher values of  $|\delta|$ , including the regions of very small  $v_2$  suggested by neutrino mass values, the lighter state  $h^0$  remains overwhelmingly doublet-dominated, and its signatures are identical to that of the SM Higgs boson. A somewhat more striking feature, however, presents itself below the cross-over region (*i. e.*, for  $|\delta| \leq 4$  for the left panel and  $|\delta| \leq 2.8$  for the right panel of figure 4). Although such an  $a_0$  is on the lower side of the electroweak scale, it is still allowed, perhaps with a mild degree of fine-tuning. The interesting point to note is that  $h^0$  is dominantly a triplet here, and, as seen from the left panels of figures 4(a), 4(b) and 4(c), it has rather suppressed interactions with both fermion and gauge boson pairs. As a result, its production in all of the usually expected channels will be suppressed. At the LHC, the gluon-gluon fusion channel will suffer due to the feeble coupling of the lightest scalar to the top quark in the loop, while the feeble character of gauge boson pair coupling will suppress production via gauge boson fusion. (However,  $H^0$  may be produced, depending on its mass.) In  $e^+e^-$  collision, too, both the processes of associated production of  $h^0$  with  $Z$  as well as gauge boson fusion will undergo suppression in an identical manner. Consequently, the experimental limit on the mass of such a scalar is much more relaxed than in the case of a Higgs doublet [22–25].

## 6 Summary and conclusions

We have considered the inclusion of a single complex Higgs triplet  $X$  in addition to the usual Higgs doublet  $\Phi$  of the standard model, motivated by the fact that this is the most economic, model-independent way of generating neutrino masses through triplet interactions. Then we have considered the most general scalar potential of such a scenario, including a term  $a_0 \Phi \Phi X^\dagger$ . We show that, with just one triplet added, such a term must be included if one has to avoid additional Goldstone bosons in neutral sector. It is further demonstrated that  $a_0$  must be real, thus ruling out any additional source of CP-violation. We have also obtained the field content requirement of a general model (with doublets, complex-triplets, real-triplets) for having additional sources of CP-violation in the tree-level potential.

We have gone on to examine the implications of the above trilinear term in the mass matrices of the neutral scalars and pseudoscalars, as also for the singly- and doubly-charged scalar masses

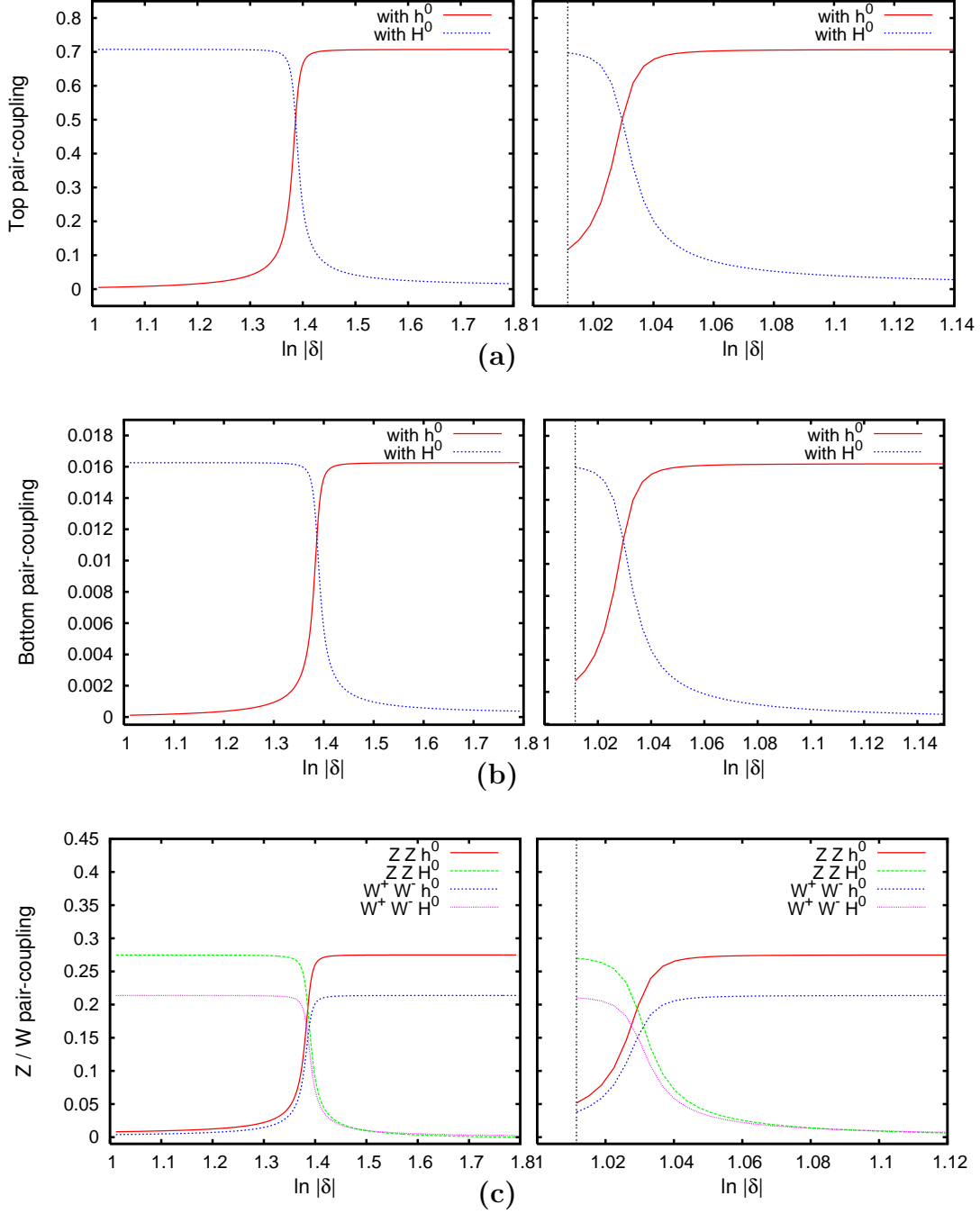


Figure 4:  $t$ ,  $b$ ,  $W^\pm$  and  $Z$  pair couplings with  $H^0$  and  $h^0$  against  $\ln|\delta|$ , with  $\lambda_i$  ( $i = 1 - 5$ ) = 1 and  $v_2 = 1$  GeV (in the left panel) and  $\lambda_1 = 0.7$ ,  $\lambda_i$  ( $i = 2 - 5$ ) = 1 and  $v_2 = 1$  GeV (in the right panel). Below the cross-over point,  $\ln|\delta| \simeq 1.39$  for left panel and  $\ln|\delta| \simeq 1.03$  for right panel, interactions of  $h^0$  are suppressed, relaxing its experimental mass limit. The regions on the left of the vertical line shown in the figures in the right panel are disallowed, by the chosen lower limits on the scalar masses, such as,  $M_{H^{++}} > 150$  GeV,  $M_{H^+}$ ,  $M_{A^0}$ ,  $M_{H^0} > 100$  GeV and  $M_{h^0} > 115$  GeV.



in this scenario. The  $a_0$ -dependent lower limits on the charged Higgs mass are derived from bounds on  $\rho$ -parameter.

We find that, for small values of  $|\delta| = |a_0|/v_2$  (where  $v_2$  is the triplet vev), the spectrum is of such nature as to allow the decay of heavier scalars into lighter ones via gauge interactions. For large  $|\delta|$ , on the other hand, the doubly-charged, singly-charged and neutral pseudoscalar Higgses become practically degenerate, while the two even-parity neutral scalars are considerably lighter. This also emphasizes the possibility of the decay of the singly-charged or neutral pseudoscalar states into the neutral scalars.

The couplings of the various physical states in this scenario has been studied in detail. It is found that, for small values of  $|\delta|$ , the lightest neutral scalar field is dominated by triplet contributions, and as such has extremely suppressed interactions with fermion as well as gauge boson pairs.

## Acknowledgments

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## Appendix: Gauge and fermion couplings with scalars

Here we present a complete list of Feynman rules for couplings of the physical scalars with gauge bosons and fermions. We define  $c_W = \cos \theta_W$  etc., and the mixing angles,

$$c_+ = \frac{v_1}{\sqrt{v_1^2 + 4v_2^2}}, \quad s_+ = \frac{2v_2}{\sqrt{v_1^2 + 4v_2^2}} \quad (30)$$

$$c_I = \frac{v_1}{\sqrt{v_1^2 + 8v_2^2}}, \quad s_I = \frac{2\sqrt{2}v_2}{\sqrt{v_1^2 + 8v_2^2}}, \quad (31)$$

for the charged scalars and pseudo scalars with their respective gauge states. For the real scalars we use the corresponding notations  $c_R$  and  $s_R$ . In terms of the elements of  $\mathcal{M}^{0R}$  in equation (16), and its eigenvalues  $\Lambda_{h^0} = 2M_{h^0}^2$  and  $\Lambda_{H^0} = 2M_{H^0}^2$ , the expression for  $c_R$  and  $s_R$  can be given as,

$$\begin{aligned} c_R &= \mathcal{M}_{12}^{0R} / \left( \sqrt{[\mathcal{M}_{12}^{0R}]^2 + [\Lambda_{h^0} - \mathcal{M}_{11}^{0R}]^2} \right) \\ s_R &= \mathcal{M}_{12}^{0R} / \left( \sqrt{[\mathcal{M}_{12}^{0R}]^2 + [\Lambda_{H^0} - \mathcal{M}_{11}^{0R}]^2} \right), \end{aligned} \quad (32)$$

which we estimate numerically. While deducing the Feynman rules, we assume all momenta (expressed in general as  $P_\mu$ ) to be incoming.

Following the above notations we now list the rules for the gauge-scalar three-vertices;

$$ZH^{++}H^{--} : i \left( g \frac{\cos 2\theta_W}{c_W} \right) [P_\mu(H^{--}) - P_\mu(H^{++})] \quad (33)$$

$$AH^{++}H^{--} : i(2e) [P_\mu(H^{--}) - P_\mu(H^{++})] \quad (34)$$

$$W^-W^-H^{++} : i(2g^2v_2) g_{\alpha\beta} \quad (35)$$

$$ZZh^0 : i \left( \frac{g^2}{c_W^2} \right) \left[ \frac{v_1 c_R}{2} + 2\sqrt{2}v_2 s_R \right] g_{\alpha\beta} \quad (36)$$

$$ZZH^0 : i \left( \frac{g^2}{c_W^2} \right) \left[ -\frac{v_1 s_R}{2} + 2\sqrt{2}v_2 c_R \right] g_{\alpha\beta} \quad (37)$$

$$W^+W^-h^0 : ig^2 \left[ \frac{v_1 c_R}{2} + 2\sqrt{2}v_2 s_R \right] g_{\alpha\beta} \quad (38)$$

$$W^+W^-H^0 : ig^2 \left[ -\frac{v_1 s_R}{2} + 2\sqrt{2}v_2 c_R \right] g_{\alpha\beta} \quad (39)$$

$$ZW^-G^+ : \left( \frac{g^2}{c_W} [\cos 2\theta_W - 1] \frac{v_1 c_+}{4} - \left[ eg' + \frac{g^2}{c_W} \right] v_2 s_+ \right) g_{\alpha\beta} \quad (40)$$

$$ZW^-H^+ : \left( -\frac{g^2}{c_W} [\cos 2\theta_W - 1] \frac{v_1 s_+}{4} - \left[ eg' + \frac{g^2}{c_W} \right] v_2 c_+ \right) g_{\alpha\beta} \quad (41)$$

$$AW^-G^+ : eg \left( \frac{v_1 c_+}{4} + v_2 s_+ \right) g_{\alpha\beta} \quad (42)$$

$$AW^-H^+ : eg \left( -\frac{v_1 s_+}{4} + v_2 c_+ \right) g_{\alpha\beta} \quad (43)$$

$$ZG^+G^- : i \left( g \frac{\cos 2\theta_w}{2c_w} c_+^2 + g' s_w s_+^2 \right) [P_\mu(G^-) - P_\mu(G^+)] \quad (44)$$

$$ZG^+H^- : i \left( -g \frac{\cos 2\theta_w}{2c_w} + g' s_w \right) c_+ s_+ [P_\mu(G^-) - P_\mu(G^+)] \quad (45)$$

$$ZH^+H^- : i \left( g \frac{\cos 2\theta_w}{2c_w} s_+^2 + g' s_w c_+^2 \right) [P_\mu(G^-) - P_\mu(G^+)] \quad (46)$$

$$AG^+G^- : ie [P_\mu(G^-) - P_\mu(G^+)] \quad (47)$$

$$AH^+H^- : ie [P_\mu(H^-) - P_\mu(H^+)] \quad (48)$$

$$Zh^0G^0 : \frac{g}{c_w} (c_R c_I / 2 + s_R s_I) [P_\mu(h^0) - P_\mu(G^0)] \quad (49)$$

$$Zh^0A^0 : \frac{g}{c_w} (s_R c_I / 2 - c_R s_I) [P_\mu(h^0) - P_\mu(A^0)] \quad (50)$$

$$ZH^0G^0 : \frac{g}{c_w} (-c_R s_I / 2 + s_R c_I) [P_\mu(H^0) - P_\mu(G^0)] \quad (51)$$

$$ZH^0A^0 : \frac{g}{c_w} (-s_R s_I / 2 - c_R c_I) [P_\mu(H^0) - P_\mu(A^0)] \quad (52)$$

$$W^-G^-H^{++} : i (g s_+) [P_\mu(G^-) - P_\mu(H^{++})] \quad (53)$$

$$W^-H^-H^{++} : i (g c_+) [P_\mu(H^-) - P_\mu(H^{++})] \quad (54)$$

$$W^-G^+h^0 : ig \left( c_+ c_R / 2 + s_+ s_R / \sqrt{2} \right) [P_\mu(h^0) - P_\mu(G^+)] \quad (55)$$

$$W^-H^+h^0 : ig \left( -s_+ c_R / 2 + c_+ s_R / \sqrt{2} \right) [P_\mu(h^0) - P_\mu(H^+)] \quad (56)$$

$$W^-G^+H^0 : ig \left( -c_+ s_R / 2 + s_+ c_R / \sqrt{2} \right) [P_\mu(H^0) - P_\mu(G^+)] \quad (57)$$

$$W^-H^+H^0 : ig \left( s_+ s_R / 2 + c_+ c_R / \sqrt{2} \right) [P_\mu(H^0) - P_\mu(H^+)] \quad (58)$$

$$W^-G^+G^0 : g \left( c_+ c_I / 2 + s_+ s_I / \sqrt{2} \right) [P_\mu(G^0) - P_\mu(G^+)] \quad (59)$$

$$W^-H^+G^0 : g \left( s_+ c_I / 2 - c_+ s_I / \sqrt{2} \right) [P_\mu(G^0) - P_\mu(H^+)] \quad (60)$$

$$W^-G^+A^0 : g \left( -c_+ s_I / 2 + s_+ c_I / \sqrt{2} \right) [P_\mu(A^0) - P_\mu(G^+)] \quad (61)$$

$$W^-H^+A^0 : g \left( -s_+ s_I / 2 - c_+ c_I / \sqrt{2} \right) [P_\mu(A^0) - P_\mu(H^+)] \quad (62)$$

The Feynman rules for gauge-scalar four-vertices are,

$$ZZH^{++}H^{--} : i \left( 2g^2 \frac{\cos^2 2\theta_w}{c_w^2} \right) g_{\alpha\beta} \quad (63)$$

$$AZH^{++}H^{--} : i \left( 4eg \frac{\cos 2\theta_w}{c_w} \right) g_{\alpha\beta} \quad (64)$$

$$AAH^{++}H^{--} : i (8e^2) g_{\alpha\beta} \quad (65)$$

$$W^+W^-H^{++}H^{--} : ig^2 g_{\alpha\beta} \quad (66)$$

$$ZZG^+G^- : i \left( g^2 \frac{\cos^2 2\theta_w}{2c_w^2} c_+^2 + 2g'^2 s_w^2 s_+^2 \right) g_{\alpha\beta} \quad (67)$$

$$ZZG^+H^- : -i \left( g^2 \frac{\cos^2 2\theta_W}{c_W^2} - 2g'^2 s_W^2 \right) c_+ s_+ g_{\alpha\beta} \quad (68)$$

$$ZZH^+H^- : i \left( g^2 \frac{\cos^2 2\theta_W}{2c_W^2} s_+^2 + 2g'^2 s_W^2 c_+^2 \right) g_{\alpha\beta} \quad (69)$$

$$AAG^+G^- : i (2e^2) g_{\alpha\beta} \quad (70)$$

$$AAH^+H^- : i (2e^2) g_{\alpha\beta} \quad (71)$$

$$AZG^+G^- : i \left( eg \frac{\cos 2\theta_W}{c_W} c_+^2 - 2eg' s_W s_+^2 \right) g_{\alpha\beta} \quad (72)$$

$$AZG^+H^- : -i \left( eg \frac{\cos 2\theta_W}{c_W} - 2eg' s_W \right) c_+ s_+ g_{\alpha\beta} \quad (73)$$

$$AZH^+H^- : i \left( eg \frac{\cos 2\theta_W}{c_W} s_+^2 - 2eg' s_W c_+^2 \right) g_{\alpha\beta} \quad (74)$$

$$W^+W^-G^+G^- : i (g^2 c_+^2 / 2 + 2g^2 s_+^2) g_{\alpha\beta} \quad (75)$$

$$W^+W^-G^+H^- : i \left( -\frac{3}{2} g^2 c_+ s_+ \right) g_{\alpha\beta} \quad (76)$$

$$W^+W^-H^+H^- : i (g^2 s_+^2 / 2 + 2g^2 c_+^2) g_{\alpha\beta} \quad (77)$$

$$ZZh^0h^0 : i \left( g^2 \frac{c_R^2}{2c_W^2} + 2g^2 \frac{s_R^2}{c_W^2} \right) g_{\alpha\beta} \quad (78)$$

$$ZZh^0H^0 : i \left( g^2 \frac{3c_R s_R}{2c_W^2} \right) g_{\alpha\beta} \quad (79)$$

$$ZZH^0H^0 : i \left( g^2 \frac{s_R^2}{2c_W^2} + 2g^2 \frac{c_R^2}{c_W^2} \right) g_{\alpha\beta} \quad (80)$$

$$W^+W^-h^0h^0 : i (g^2 c_R^2 / 2 + 2g^2 s_R^2) g_{\alpha\beta} \quad (81)$$

$$W^+W^-h^0H^0 : i (g^2 c_R s_R / 2) g_{\alpha\beta} \quad (82)$$

$$W^+W^-H^0H^0 : i (g^2 s_R^2 / 2 + 2g^2 c_R^2) g_{\alpha\beta} \quad (83)$$

$$ZZG^0G^0 : i \left( g^2 \frac{c_I^2}{2c_W^2} + 2g^2 \frac{s_I^2}{c_W^2} \right) g_{\alpha\beta} \quad (84)$$

$$ZZG^0A^0 : -i \left( g^2 \frac{3c_I s_I}{2c_W^2} \right) g_{\alpha\beta} \quad (85)$$

$$ZZA^0A^0 : i \left( g^2 \frac{s_I^2}{2c_W^2} + 2g^2 \frac{c_I^2}{c_W^2} \right) g_{\alpha\beta} \quad (86)$$

$$W^+W^-G^0G^0 : i (g^2 c_I^2 / 2 + 2g^2 s_I^2) g_{\alpha\beta} \quad (87)$$

$$W^+W^-G^0A^0 : -i (g^2 c_I s_I / 2) g_{\alpha\beta} \quad (88)$$

$$W^+W^-A^0A^0 : i (g^2 s_I^2 / 2 + 2g^2 c_I^2) g_{\alpha\beta} \quad (89)$$

$$ZW^-H^{++}G^- : i \left( g^2 \frac{\cos 2\theta_W}{c_W} - gg' s_W \right) s_I g_{\alpha\beta} \quad (90)$$

$$ZW^- H^{++} H^- : i \left( g^2 \frac{\cos 2\theta_w}{c_w} - gg' s_w \right) c_I g_{\alpha\beta} \quad (91)$$

$$AW^- H^{++} G^- : i (3eg) s_I g_{\alpha\beta} \quad (92)$$

$$AW^- H^{++} H^- : i (3eg) c_I g_{\alpha\beta} \quad (93)$$

$$ZW^- G^+ h^0 : i \left( \frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{c_+ c_R}{4} - \left[ eg' + \frac{g^2}{c_w} \right] \frac{s_+ s_R}{\sqrt{2}} \right) g_{\alpha\beta} \quad (94)$$

$$ZW^- G^+ H^0 : i \left( -\frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{c_+ s_R}{4} - \left[ eg' + \frac{g^2}{c_w} \right] \frac{s_+ c_R}{\sqrt{2}} \right) g_{\alpha\beta} \quad (95)$$

$$ZW^- H^+ h^0 : i \left( -\frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{s_+ c_R}{4} - \left[ eg' + \frac{g^2}{c_w} \right] \frac{c_+ s_R}{\sqrt{2}} \right) g_{\alpha\beta} \quad (96)$$

$$ZW^- H^+ H^0 : i \left( \frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{s_+ s_R}{4} - \left[ eg' + \frac{g^2}{c_w} \right] \frac{c_+ c_R}{\sqrt{2}} \right) g_{\alpha\beta} \quad (97)$$

$$AW^- G^+ h^0 : i \left( eg \left[ \frac{c_+ c_R}{2} + \frac{s_+ s_R}{\sqrt{2}} \right] \right) g_{\alpha\beta} \quad (98)$$

$$AW^- G^+ H^0 : i \left( eg \left[ -\frac{c_+ s_R}{2} + \frac{s_+ c_R}{\sqrt{2}} \right] \right) g_{\alpha\beta} \quad (99)$$

$$AW^- H^+ h^0 : i \left( eg \left[ -\frac{s_+ c_R}{2} + \frac{c_+ s_R}{\sqrt{2}} \right] \right) g_{\alpha\beta} \quad (100)$$

$$AW^- H^+ H^0 : i \left( eg \left[ \frac{s_+ s_R}{2} + \frac{s_+ c_R}{\sqrt{2}} \right] \right) g_{\alpha\beta} \quad (101)$$

$$ZW^- G^+ G^0 : \left( \frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{c_+ c_I}{4} - \left[ eg' + \frac{g^2}{c_w} \right] \frac{s_+ s_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (102)$$

$$ZW^- G^+ A^0 : \left( \frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{c_+ s_I}{4} + \left[ eg' + \frac{g^2}{c_w} \right] \frac{s_+ c_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (103)$$

$$ZW^- H^+ G^0 : \left( -\frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{s_+ c_I}{4} - \left[ eg' + \frac{g^2}{c_w} \right] \frac{c_+ s_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (104)$$

$$ZW^- H^+ A^0 : \left( -\frac{g^2}{c_w} [\cos 2\theta_w - 1] \frac{s_+ s_I}{4} + \left[ eg' + \frac{g^2}{c_w} \right] \frac{c_+ c_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (105)$$

$$AW^- G^+ G^0 : eg \left( \frac{c_+ c_I}{2} + \frac{s_+ s_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (106)$$

$$AW^- G^+ A^0 : eg \left( \frac{c_+ s_I}{2} - \frac{s_+ c_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (107)$$

$$AW^- H^+ G^0 : eg \left( -\frac{s_+ c_I}{2} + \frac{c_+ s_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (108)$$

$$AW^- H^+ A^0 : eg \left( -\frac{s_+ s_I}{2} - \frac{c_+ c_I}{\sqrt{2}} \right) g_{\alpha\beta} \quad (109)$$

$$W^+ W^+ H^{--} h^0 : i \left( \sqrt{2} g^2 s_R \right) g_{\alpha\beta} \quad (110)$$

$$W^+ W^+ H^{--} H^0 : i \left( \sqrt{2} g^2 c_R \right) g_{\alpha\beta} \quad (111)$$

$$W^+W^+H^{--}G^0 \quad : \quad -\left(\sqrt{2}g^2s_I\right)g_{\alpha\beta} \quad (112)$$

$$W^+W^+H^{--}A^0 \quad : \quad \left(\sqrt{2}g^2c_I\right)g_{\alpha\beta}. \quad (113)$$

Feynman rules for the fermion-scalar vertices are,

$$G^+\bar{u}_id_j \quad : \quad \frac{ig}{\sqrt{2}M_W} [m_{u_i}P_L - m_{d_j}P_R] V_{ij} \quad (114)$$

$$G^-\bar{d}_ju_i \quad : \quad \frac{ig}{\sqrt{2}M_W} [m_{u_i}P_R - m_{d_j}P_L] V_{ij}^* \quad (115)$$

$$H^+\bar{u}_id_j \quad : \quad \frac{igs_+}{\sqrt{2}M_Wc_+} [m_{u_i}P_L - m_{d_j}P_R] V_{ij} \quad (116)$$

$$H^-\bar{d}_ju_i \quad : \quad \frac{igs_+}{\sqrt{2}M_Wc_+} [m_{u_i}P_R - m_{d_j}P_L] V_{ij}^* \quad (117)$$

$$G^0\bar{u}_iu_i \quad : \quad -\frac{gc_I}{2M_Wc_+}m_{u_i}\gamma_5 \quad (118)$$

$$G^0\bar{d}_id_i \quad : \quad \frac{gc_I}{2M_Wc_+}m_{d_i}\gamma_5 \quad (119)$$

$$A^0\bar{u}_iu_i \quad : \quad -\frac{gs_I}{2M_Wc_+}m_{u_i}\gamma_5 \quad (120)$$

$$A^0\bar{d}_id_i \quad : \quad \frac{gs_I}{2M_Wc_+}m_{d_i}\gamma_5 \quad (121)$$

$$h^0\bar{u}_iu_i \quad : \quad \frac{igc_R}{2M_Wc_+}m_{u_i} \quad (122)$$

$$h^0\bar{d}_id_i \quad : \quad \frac{igc_R}{2M_Wc_+}m_{d_i} \quad (123)$$

$$H^0\bar{u}_iu_i \quad : \quad \frac{igs_R}{2M_Wc_+}m_{u_i} \quad (124)$$

$$H^0\bar{d}_id_i \quad : \quad \frac{igs_R}{2M_Wc_+}m_{d_i} \quad (125)$$

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